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S-GSTAR-SUR Model for Seasonal Spatio Temporal Data Forecasting

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ABSTRACT

Generalized Space Time Autoregressive (GSTAR) is one of space-time models that frequently used for forecasting spatio-temporal data. Up to now, the researches about GSTAR only focused on stationary non-seasonal spatio-temporal data. Ordinary Least Squares (OLS) is a method that usually applied to estimate the parameters of GSTAR model. Parameter estimation by using OLS for GSTAR model with correlated residuals between equations will produce inefficient estimators. The method that appropriate to estimate the parameter model with correlated residuals between equations is Generalized Least Square (GLS), which is usually used in Seemingly Unrelated Regression (SUR) model. The purpose of this research is to propose GLS method for estimating parameters in seasonal GSTAR models, known as S-GSTAR-SUR. Moreover, this research also proposes a spatial weight based on the normalization of partial cross-correlation inference. Simulation study is done for evaluating the efficiency of GLS estimators. Then, the number of tourist arrivals at four tourism locations in Indonesia (i.e. Jakarta, Bali, Surabaya, and Surakarta) is used as a case study. The results of simulation study show that S-GSTAR-SUR yields more efficient estimators than S-GSTAR-OLS when the residuals between equations are correlated. It is showed by the smaller standard error of S-GSTAR-SUR estimators. Additionally, the comparison of forecast accuracy between Vector Autoregressive Integrated Moving Average (VARIMA), S-GSTAR-OLS and S-GSTAR-SUR shows that S-GSTAR-SUR model with spatial weight based on normalization of partial cross-correlation inference yields the smallest RMSE for forecasting the number of tourist arrivals at four tourism locations in Indonesia.

Keywords: GSTAR, GLS, SUR, Seasonal, Spatio-temporal, Tourist Arrivals.

1. Introduction

In daily life, we often find data that not only have correlation with the events at the previous times, but also correlate to location or another space that is usually referred to as spatial data. Space-time model is a model that combines of time and location dependencies in a multivariate time series data.

One of the space-time models that frequently used is the Generalized Space-Time Autoregressive or GSTAR which is introduced by Borovkova *et al*. (2002). There are several studies that have been conducted relating to the application of GSTAR, such as Ruchjana (2002) who have applied GSTAR for petroleum production modeling. Deng and Athanasopoulos (2011) used Space-Time Autoregressive Integrated Moving Average (STARIMA) for prediction of domestic tourists in Australia, and Wutsqa and Suhartono (2010) also applied VAR-GSTAR model for forecasting the number of tourist arrivals. Moreover, Nurhayati *et al*. (2012) applied GSTAR for forecasting GDP in Western European countries.

Until now, most researches that related to GSTAR only focused on stationary and non-seasonally spatio-temporal data. In addition, Ordinary Least Squares (OLS) is a method that usually used to estimate the parameters in the GSTAR model (Borovkova *et al*. (2008)). OLS estimation in the multivariate models, including GSTAR, with correlated residuals would yield inefficient estimators. One of the appropriate estimation methods in the case of correlated residuals is Generalized Least Square (GLS), which is commonly used in the Seemingly Unrelated Regression (SUR) model (Zellner (1962); Henningsen and Hamann (2007)).

The purpose of this research is to study theoretically about the GLS method for estimating the parameters of GSTAR model, and then referred to as GSTAR-SUR. Furthermore, the results of the theoretical study will be validated on a simulation study, i.e. through the estimator comparison of GSTAR-SUR and GSTAR-OLS which is applied to seasonal data and the combination of seasonal and non-seasonal data using spatial weight based on normalization of partial cross-correlation inference. As a case study, the results of theoretical and simulation study was then applied to forecasting the number of tourist arrivals in four main gates in Indonesia, namely Jakarta, Bali, Surabaya and Surakarta. The results are compared to the forecast of VARIMA model by using RMSE criteria.

2. Methods

In this section, the statistical methods that are used for statistical estimations are presented.

2.1 VARIMA (Vector Autoregressive Integrated Moving Average) Model

Let $Z_i(t)$ with $t \in T$, $t = \{1, 2,\ldots,T)$ and $i = \{1,2,\ldots,N\}$ are index of time and variables, then the VARIMA model can be expressed as follows (Wei(2006)):

$$\mathbf{\Phi}_p(B)\mathbf{Z}(t) = \mathbf{\Theta}_q(B)\mathbf{e}(t)\cdot \tag{1}$$

VARIMA model building is done through the steps of identification, parameter estimation, diagnostic checks, and forecasting such as the Box-Jenkins procedure (Suhartono and Atok (2005)). Identification step is done for determining order of the model by using a time series plot, MCCF (Matrix Cross Correlation Function), MPCCF (Matrix Partial Cross Correlation Function), and the AIC (Akaike's Information Criterion). Parameter estimation step is done using the method of Least Square or Maximum Likelihood. Then, diagnostic checks is performed to evaluate whether the residuals of the model has been satisfied white noise condition. Finally, the best model is used to calculate the final prediction, both point and interval prediction.

2.2 GSTAR Model

GSTAR is a generalization of the STAR models. Let {Z(t) : t=0,±1,±2,...} is a multivariate time series of *N* locations, then GSTAR with time order *p* and spatial order $\lambda_1, \lambda_2, ..., \lambda_p$, i.e. GSTAR$\left(p; \lambda_1, \lambda_2, ..., \lambda_p\right)$, in matrix notation can be written as follows (Borovkova *et al*. (2008)):

$$\mathbf{Z}(t) = \sum_{s=1}^{p}\left(\mathbf{\Phi}_{s0} + \sum_{k=1}^{\lambda_s}\mathbf{\Phi}_{sk}\mathbf{W}^{(k)}\right)\mathbf{Z}(t-s) + \mathbf{e}(t) \tag{2}$$

where $\mathbf{\Phi}_{s0} = \mathrm{diag}(\phi_{10}^s, \cdots, \phi_{N0}^s)$, $\mathbf{\Phi}_{sk} = \mathrm{diag}(\phi_{1k}^s, \cdots, \phi_{Nk}^s)$, $\mathbf{e}(t)$ is residual model that satisfies identically, independent, distributed with mean $\mathbf{0}$ and covariance $\mathbf{\Sigma}$. For instance, GSTAR model with time and spatial order one for three locations is as follows:

$$\mathbf{Z}(t) = \mathbf{\Phi}_{10}\mathbf{Z}(t-1) + \mathbf{\Phi}_{11}\mathbf{W}^{(1)}\mathbf{Z}(t-1) + \mathbf{e}(t) \tag{3}$$

and in matrix form, equation (3) can be written as follows:

$$\begin{pmatrix} Z_1(t) \\ Z_2(t) \\ Z_3(t) \end{pmatrix} = \begin{pmatrix} \phi_{10} & 0 & 0 \\ 0 & \phi_{20} & 0 \\ 0 & 0 & \phi_{30} \end{pmatrix} \begin{pmatrix} Z_1(t-1) \\ Z_2(t-1) \\ Z_3(t-1) \end{pmatrix} + \begin{pmatrix} \phi_{11} & 0 & 0 \\ 0 & \phi_{21} & 0 \\ 0 & 0 & \phi_{31} \end{pmatrix} \begin{pmatrix} 0 & w_{12} & w_{13} \\ w_{21} & 0 & w_{23} \\ w_{31} & w_{32} & 0 \end{pmatrix} \begin{pmatrix} Z_1(t-1) \\ Z_2(t-1) \\ Z_3(t-1) \end{pmatrix} + \begin{pmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \end{pmatrix}. \tag{4}$$

There are several matrices of spatial weights or **W** that usually used in GSTAR model, i.e. uniform weight, weight based on inverse of distance between locations, weight based on normalization of cross correlation inference, and weight based on normalization of partial cross correlation inference (Borovkova*et al*. (2008)).

2.3 SUR (Seemingly Unrelated Regression)

SUR is a system of equations consisting of multiple regression equations where the residual is not correlated between observations in a single equation, but has correlation between the residual equations. Information about the presence of correlation between residual equations can be used by Generalized Least Square (GLS) method to improve the model estimators. GLS is an estimation method of regression parameter that consider the correlation of the residuals between equations, where the residual estimates obtained from Ordinary Least Square (OLS) will be used in the estimation of regression coefficients in the equation SUR system. In general, SUR models for N equations where each equation consists of K predictors can be written as follows:

$$\begin{aligned} Y_1 &= \beta_{10} + \beta_{11} X_{1,1} + \beta_{12} X_{1,2} + \ldots + \beta_{1K} X_{1,K} + e_1 \\ Y_2 &= \beta_{20} + \beta_{21} X_{2,1} + \beta_{22} X_{2,2} + \ldots + \beta_{2K} X_{2,K} + e_2 \\ &\vdots \qquad\qquad\qquad \vdots \\ Y_N &= \beta_{N0} + \beta_{N1} X_{N,1} + \beta_{N2} X_{N,2} + \ldots + \beta_{NK} X_{N,K} + e_N \end{aligned} \tag{5}$$

where i = 1, 2,…,N.

The assumptions that must be fulfilled at the SUR model are $E(\boldsymbol{\varepsilon}) = \mathbf{0}$, and $E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}') = \boldsymbol{\sigma}_{ij}\mathbf{I}_T$, where i, j = 1, 2,…,N.

2.4 Criteria for Selection the Best Model

The best model is selected based on the prediction accuracy of the out sample data. The criteria are out sample RMSE, which means that the best model is the model that has the smallest RMSE. The formula to calculate RMSE at out sample data is as follows:

$$RMSE = \sqrt{\frac{1}{M}\sum_{l=1}^{M}\left(Z_{T+l} - \hat{Z}_T(l)\right)^2} \cdot \tag{6}$$

3. Results

In this section, we firstly present the theoretical results, then the results of simulation study, and finally the results of empirical study.

3.1 Estimator $\hat{\boldsymbol{\beta}}$ of Seasonal GSTAR-SUR Model

Let $\{\mathbf{Z}(t): t = 0,\pm1,\pm2,\ldots\}$ is a multivariate time series of N locations, then the GSTAR($[12]_1$) model can be written as

$$\mathbf{Z}(t) = \left(\mathbf{\Phi}_0^{12} + \mathbf{\Phi}_1^{12}\mathbf{W}\right)\mathbf{Z}(t-12) + \mathbf{e}(t) \tag{7}$$

where time parameter $\mathbf{\Phi}_0^{12}$ and spatial parameter $\mathbf{\Phi}_1^{12}$ and spatial weight $\mathbf{W}$ as follows:

$$\mathbf{\Phi}_0^{12} = \begin{bmatrix} \phi_{10}^{12} & 0 & \cdots & 0 \\ 0 & \phi_{20}^{12} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \phi_{N0}^{12} \end{bmatrix}, \mathbf{\Phi}_1^{12} = \begin{bmatrix} \phi_{11}^{12} & 0 & \cdots & 0 \\ 0 & \phi_{21}^{12} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \phi_{N1}^{12} \end{bmatrix}, \mathbf{W} = \begin{bmatrix} 0 & w_{12} & \cdots & w_{1N} \\ w_{21} & 0 & \cdots & w_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ w_{N1} & w_{N2} & \cdots & 0 \end{bmatrix}.$$

Then, model in Eq. (7) could be written in matrix form as follows:

$$\begin{bmatrix} \mathbf{Z}_1(t) \\ \mathbf{Z}_2(t) \\ \mathbf{Z}_3(t) \\ \vdots \\ \mathbf{Z}_N(t) \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_1(t-12) & \mathbf{V}_1(t-12) & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{Z}_N(t-12) & \mathbf{V}_N(t-12) \end{bmatrix} \begin{bmatrix} \phi_{10}^{12} \\ \phi_{11}^{12} \\ \vdots \\ \phi_{N0}^{12} \\ \phi_{N1}^{12} \end{bmatrix} + \begin{bmatrix} \mathbf{e}_1(t) \\ \mathbf{e}_2(t) \\ \mathbf{e}_3(t) \\ \vdots \\ \mathbf{e}_N(t) \end{bmatrix} \tag{8}$$

where

$$\mathbf{Z}_i(t) = \begin{bmatrix} z_i(1) \\ z_i(2) \\ \vdots \\ z_i(T) \end{bmatrix}, \mathbf{Z}_i(t-12) = \begin{bmatrix} z_i(-11) \\ z_i(-10) \\ \vdots \\ z_i(T-12) \end{bmatrix}, \mathbf{e}_i(t) = \begin{bmatrix} e_i(1) \\ e_i(2) \\ \vdots \\ e_i(T) \end{bmatrix}, \mathbf{V}_i(t-12) = \begin{bmatrix} \sum_{j\neq i} w_{ij} z_j(-11) \\ \sum_{j\neq i} w_{ij} z_j(-10) \\ \vdots \\ \sum_{j\neq i} w_{ij} z_j(T-12) \end{bmatrix}$$

and i= 1, 2,…, N. Thus, for each i, we have equation

$$\mathbf{Z}_i(t) = \phi_{i0}^{12}\mathbf{Z}_i(t-12) + \phi_{i1}^{12}\mathbf{V}_i(t-12) + \mathbf{e}_i(t) \cdot \tag{9}$$

The equation of GSTAR-SUR model could be written as follows:

$$\mathbf{Y}_{i,t} = \mathbf{X}_{i,t}\,\boldsymbol{\beta}_i + \boldsymbol{\varepsilon}_{i,t} \tag{10}$$

where :

$$\mathbf{Y}_{i,t} = \mathbf{Z}_i(t), \quad \mathbf{X}_{i,t} = \left[\mathbf{Z}_i(t-12) \quad \mathbf{V}_i(t-12)\right], \quad \boldsymbol{\varepsilon}_{i,t} = \mathbf{e}_i(t), \quad \boldsymbol{\beta}_i = \begin{bmatrix} \phi_{i0}^{12} \\ \phi_{i1}^{12} \end{bmatrix}.$$

Thus, the equation of GSTAR-SUR model in matrix representation could be written as follows:

$$\begin{array}{ccccccc} \begin{bmatrix} \mathbf{Y}_{1,t} \\ \mathbf{Y}_{2,t} \\ \vdots \\ \mathbf{Y}_{N,t} \end{bmatrix} & = & \begin{bmatrix} \mathbf{X}_{1,t} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_{2,t} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{X}_{N,t} \end{bmatrix} & \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_N \end{bmatrix} & + & \begin{bmatrix} \boldsymbol{\varepsilon}_{1,t} \\ \boldsymbol{\varepsilon}_{2,t} \\ \vdots \\ \boldsymbol{\varepsilon}_{N,t} \end{bmatrix} & . \\ \mathbf{Y} & = & \mathbf{X} & \boldsymbol{\beta} & + & \boldsymbol{\varepsilon} & \end{array} \tag{11}$$

The residuals of GSTAR-SUR model are assumed not correlated in each location *i*, i.e.

$$E\left(\varepsilon_{i,t}\,\varepsilon_{j,s}\right) = \begin{cases} 0, \; t \neq s \\ \sigma_{ij}, \; t = s \end{cases}$$

where $i, j = 1, 2,\ldots, N$ and $t, s = 1, 2,\ldots, T$. But, the residuals of GSTAR-SUR model are correlated among equations or locations. Hence, variance-covariance matix of residual is

$$E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}') = \boldsymbol{\sigma}_{ij}\mathbf{I}_T$$

Since $E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}') = \boldsymbol{\sigma}_{ij}\mathbf{I}_T$ then

$$E\left(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}'\right) = \begin{bmatrix} \sigma_{11}\mathbf{I}_T & \sigma_{12}\mathbf{I}_T & \cdots & \sigma_{1N}\mathbf{I}_T \\ \sigma_{21}\mathbf{I}_T & \sigma_{22}\mathbf{I}_T & \cdots & \sigma_{2N}\mathbf{I}_T \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1}\mathbf{I}_T & \sigma_{N2}\mathbf{I}_T & \cdots & \sigma_{NN}\mathbf{I}_T \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_{NN} \end{bmatrix} \otimes \mathbf{I_T} = \boldsymbol{\Sigma} \otimes \mathbf{I_T} = \boldsymbol{\Omega}$$

where $\boldsymbol{\Omega}$ is matrix $(N\times T)\times(N\times T)$.

Parameter estimation in GSTAR-SUR model is done by applying Generalized Least Square (GLS) method, i.e. by minimizing generalized sum of square $\boldsymbol{\varepsilon}'\boldsymbol{\Omega}^{-1}\boldsymbol{\varepsilon}$. The results of GLS estimators of seasonal GSTAR-SUR model are as follows:

$$\hat{\boldsymbol{\beta}} = \left(\mathbf{X}'\boldsymbol{\Omega}^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\boldsymbol{\Omega}^{-1}\mathbf{Y}\,\cdot$$

Since $\boldsymbol{\Omega} = \boldsymbol{\Sigma} \otimes \mathbf{I_T}$, then $\boldsymbol{\beta}$ estimator is

$$\hat{\boldsymbol{\beta}} = \left(\mathbf{X}'(\boldsymbol{\Sigma} \otimes \mathbf{I_T})^{-1}\mathbf{X}\right)^{-1} \mathbf{X}'(\boldsymbol{\Sigma} \otimes \mathbf{I_T})^{-1}\mathbf{Y} \cdot \tag{12}$$

The asymptotics properties of GLS estimators are as follows:

(a) If $E(\mathbf{Y}) = \mathbf{X}\boldsymbol{\beta}$, then $\hat{\boldsymbol{\beta}}$ is unbiased estimator for $\boldsymbol{\beta}$, i.e.

$$E(\hat{\boldsymbol{\beta}}) = \boldsymbol{\beta}$$

If $\text{cov}(\mathbf{Y}) = \boldsymbol{\Omega}$, then variance-covariance matrix of $\hat{\boldsymbol{\beta}}$ is

$$\text{Cov}\left(\hat{\boldsymbol{\beta}}\right) = \left(\mathbf{X}'\boldsymbol{\Omega}^{-1}\mathbf{X}\right)^{-1}$$

(b) If $\boldsymbol{\varepsilon}$ follow normal distribution with mean $\mathbf{0}$ and variance $\boldsymbol{\sigma}_{ij}\mathbf{I}_T$, and write in matrix representation as

$$\varepsilon \sim N\left(\mathbf{0}, \boldsymbol{\sigma}_{ij}\mathbf{I}_T\right)$$

then estimator $\hat{\boldsymbol{\beta}}$ is asymptotics normal distribution with mean $\boldsymbol{\beta}$ and variance-covariance matrix $\text{Cov}(\hat{\boldsymbol{\beta}})$, so

$$\hat{\boldsymbol{\beta}} \sim N\left(\boldsymbol{\beta}, \text{Cov}\left(\hat{\boldsymbol{\beta}}\right)\right)$$

where:

$$\boldsymbol{\beta} = \begin{bmatrix} \phi_{10}^{12} \\ \phi_{11}^{12} \\ \vdots \\ \phi_{N0}^{12} \\ \phi_{N1}^{12} \end{bmatrix}, \ \text{cov}\left(\hat{\boldsymbol{\beta}}\right) = \begin{bmatrix} \text{var}\left(\phi_{10}^{12}\right) & \text{cov}\left(\phi_{10}^{12}, \phi_{11}^{12}\right) & \cdots & \text{cov}\left(\phi_{10}^{12}, \phi_{N0}^{12}\right) & \text{cov}\left(\phi_{10}^{12}, \phi_{N1}^{12}\right) \\ \text{cov}\left(\phi_{11}^{12}, \phi_{10}^{12}\right) & \text{var}\left(\phi_{11}^{12}\right) & \cdots & \text{cov}\left(\phi_{11}^{12}, \phi_{N0}^{12}\right) & \text{cov}\left(\phi_{11}^{12}, \phi_{N1}^{12}\right) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \text{cov}\left(\phi_{N0}^{12}, \phi_{10}^{12}\right) & \text{cov}\left(\phi_{N0}^{12}, \phi_{11}^{12}\right) & \cdots & \text{var}\left(\phi_{N0}^{12}\right) & \text{cov}\left(\phi_{N0}^{12}, \phi_{N1}^{12}\right) \\ \text{cov}\left(\phi_{N1}^{12}, \phi_{10}^{12}\right) & \text{cov}\left(\phi_{N1}^{12}, \phi_{11}^{12}\right) & \cdots & \text{cov}\left(\phi_{N1}^{12}, \phi_{N0}^{12}\right) & \text{var}\left(\phi_{N1}^{12}\right) \end{bmatrix}.$$

Hence, each element of $\hat{\boldsymbol{\beta}}$ is normal distribution, i.e.

$$\begin{aligned} \left(\hat{\phi}_{i0}^{12} - \phi_{i0}^{12}\right) &\sim N\left(0, \text{var}\left(\hat{\phi}_{i0}^{12}\right)\right) \\ \left(\hat{\phi}_{i1}^{12} - \phi_{i1}^{12}\right) &\sim N\left(0, \text{var}\left(\hat{\phi}_{i1}^{12}\right)\right). \end{aligned} \tag{13}$$

3.2 Modeling of Seasonal Simulation Data, and Combination Seasonal and Non Seasonal Simulation Data using GSTAR-OLS and GSTAR-SUR

The results of a simulation study using both the seasonal and the combination between seasonal and non-seasonal data, show that the optimal spatial weight GSTAR is found by normalizing the results of statistical inference on the partial cross-correlation between the locations of the corresponding time lag.

Moreover, the simulation results also show that the estimator of GSTAR-SUR model is more efficient than GSTAR-OLS models when the residual between locations are correlated. It is shown by the smaller standard error of GSTAR-SUR estimators.

3.3 Modeling of Tourist Arrivals Data using VARIMA Model

In this research, the data used as empirical study are secondary data about the number of tourist arrivals to Jakarta, Bali, Surabaya and Surakarta, which is obtained from the Indonesian Central Bureau of Statistics. The data is from January 1996 to December 2013 and divided into in sample and out sample data, i.e. data from January 1996 to December of 2011 as in sample data, while the out sample data is starting from January 2012 to December 2013.

The identification step of VARIMA model shows that data are not stationer in mean both at seasonal and non-seasonal pattern. Hence, differencing on both non seasonal (d=1) and seasonal (D=1, S=12) order are applied to make stationary data. Then, AIC and MPCCF plot are used to identify the temporary order of VARIMA model. The MPCCF plot shows that the significance lags are at lag 1 and 12, whereas the smallest AIC is at AR(2) and MA(0). Thus, the temporary VARIMA model based on both AIC and MPCCF plot is VARIMA$([1,2,12],1,0)(0,1,0)^{12}$. The results of parameter estimation and diagnostic check show that all parameters are statistically significance and residuals model satisfy the white noise condition. Hence, VARIMA$([1,2,12],1,0)(0,1,0)^{12}$ is appropriate model for forecasting these tourist arrivals data.

3.4 Modeling of Tourist Arrivals Data using GSTAR-OLS Model

OLS method is used to estimate the parameters of GSTAR-OLS model. Three spatial weights are applied for GSTAR modeling, i.e. uniform weight, weight based on inverse of distance, and weight based on normalization of partial cross correlation inference. The time lag order of GSTAR-OLS model is determined based on the order of previous VARIMA model, i.e. involve lag 1, 2 and 12. Whereas, the spatial order of GSTAR-OLS model is assumed following the 1st spatial order. Hence, the GSTAR-OLS model that be used in this analysis is GSTAR$([1,2,12]_1)$-I$(1)(1)^{12}$.

The results of parameter estimation of GSTAR-OLS model using uniform and inverse of distance weight yield 10 statistically significance parameters.

Whereas, GSTAR-OLS model with spatial weight based on normalization of partial cross correlation inference gives 12 parameters is statistically significance. Moreover, the diagnostic check shows that the residuals of these GSTAR-OLS models fulfill white noise assumption.

3.5 Modeling of Tourist Arrivals Data using GSTAR-SUR Model

In this research, the same spatial and time order model is used in VARIMA and GSTAR models, both GSTAR-OLS and GSTAR-SUR, i.e. $([1,2,12]_1$-$I(1)(1)^{12}$. Furthermore, the spatial weights in both GSTAR-SUR and GSTAR-OLS are also similar, namely uniform weight, weight based on inverse of distance, and weight based on normalization of partial cross correlation inference. The estimation method in GSTAR-SUR model is Generalized Least Squares or GLS which is usually used for SUR model.

As in GSTAR-OLS model, the results of parameter estimation of GSTAR-SUR model using uniform and inverse of distance weight also yield 10 statistically significance parameters. Moreover, GSTAR-SUR model with both spatial weights give the same parameter estimate values. Otherwise, the results also show that the standard errors of GSTAR-SUR estimators are smaller than GSTAR-OLS estimators. It proves that GSTAR-SUR yield more efficient estimator than GSTAR-OLS. The empirical result of this comparison is shown at Table 1.

TABLE 1: The comparison between standard error of estimator from GSTAR-OLS and GSTAR-SUR model using uniform and inverse of distance weight

Parameter	GSTAR-OLS		GSTAR-SUR	
	Coefficient value	**Standard Error**	**Coefficient value**	**Standard Error**
ϕ_{10}^{1}	-0.374	0.0689	-0.432	0.0646
ϕ_{10}^{12}	-0.267	0.0723	-0.312	0.0679
ϕ_{20}^{1}	-0.119	0.0667	-0.148	0.0640
ϕ_{10}^{12}	-0.510	0.0685	-0.523	0.0658
ϕ_{30}^{1}	-0.291	0.0691	-0.276	0.0666
ϕ_{30}^{2}	-0.227	0.0689	-0.187	0.0664
ϕ_{30}^{12}	-0.387	0.0660	-0.403	0.0637
ϕ_{40}^{1}	-0.255	0.0724	-0.255	0.0718
ϕ_{40}^{2}	-0.294	0.0772	-0.289	0.0765
ϕ_{40}^{12}	-0.306	0.0774	-0.297	0.0766

Furthermore, the result of GSTAR-SUR estimator using spatial weight based on normalization of partial cross correlation inference also shows that 12 parameter are statistically significance. As at the previous results, it also shows that the standard errors of GSTAR-SUR estimators are smaller than GSTAR-OLS estimators. Hence, this empirical result proves consistently that GSTAR-SUR model yield more efficient estimator than GSTAR-OLS model.

3.6 The Comparison of Forecast Accuracy between VARIMA, GSTAR-OLS and GSTAR-SUR for Tourist Arrivals Prediction

Forecasting of the number of tourist arrivals was done by applying seven previous models, i.e. VARIMA, both GSTAR-OLS and GSTAR-SUR with three spatial weights (uniform, inverse of distance, and normalization of partial cross correlation inference). The RMSE of each model is shown at Table 3.

TABLE 3: The results of forecast accuracy comparison between VARIMA, GSTAR-OLS and GSTAR-SUR model

Model	RMSE at Out-Sample				RMSE Total
	Jakarta	Bali	Surabaya	Surakarta	
VARIMA	22,201	**16,391***	1,359	1,131	13,827
GSTAR-OLS					
- uniform weight	17,843	20,808	1,198	**462***	13,720
- inverse of distance weight	17,843	20,808	1,198	**462***	13,720
- normalization of PCC inference weight	**17,601***	20,808	1,329	**462***	13,645
GSTAR-SUR					
- uniform weight	17,726	20,614	**1,193***	463	13,609
- inverse of distance weight	17,726	20,614	**1,193***	463	13,608
- normalization of PCC inference weight	17,611	20,681	1,325	463	**13,600***

In total, it shows that GSTAR-SUR with spatial weight based on normalization of partial cross correlation inference yield the smallest RMSE, i.e. 13.600, and it also means that this model is the best forecasting model for tourist arrivals prediction. Additionally, Figure 1 shows the forecasting of the number of tourist arrivals at out sample data by using GSTAR-SUR.

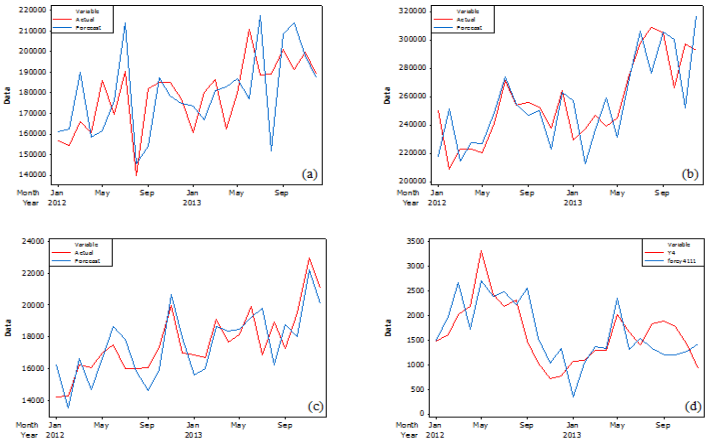

Figure 1: The comparison between actual and forecast at out sample data using GSTAR-SUR, i.e. number of tourist arrivals to Jakarta (a), Bali (b), Surabaya (c), and Surakarta (d)

4. Conclusion

Based on the results at the previous section, it can be concluded that Generalized Least Squares (GLS) method could be straightforwardly applied in GSTAR model for estimating the parameters, particularly when the residuals between locations are correlated. This research already proposed the GSTAR model with GLS estimation, then known as GSTAR-SUR, and how to construct data structure for calculating the estimators.

Furthermore, the results of simulation study that focused to seasonal and combination between seasonal and non-seasonal models showed that the determination of spatial weight on GSTAR model could be optimally done by applying a normalization of statistical inference on the partial cross-correlation between the locations of the corresponding time lag. Additionally, the simulation study also proved that GSTAR-SUR yield more efficient estimators than GSTAR-OLS when the residuals between locations are correlated. It was shown by the smaller standard error of the GSTAR-SUR estimators. Moreover, the empirical results showed that GSTAR-SUR with spatial weight based on normalization of partial cross-correlation inference gave more accurate forecast than GSTAR-OLS and VARIMA models for tourist arrivals prediction.

In addition, this research is limited to GSTAR model without predictor. Hence, further research is needed to develop GSTAR models which contain predictor as in ARIMAX or VARIMAX, both of metric and nonmetric predictor.

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